



TOPIC #1:

**FACTORING POLYNOMIALS
WITH COMMON
MONOMIAL FACTOR**

The background is a stylized illustration of a notebook page. It features a red grid pattern in the top left, a pencil on the right, and various mathematical diagrams including a triangle with a shaded area, a cylinder, and a rectangular prism. A handwritten equation $x = \frac{1}{2}y - z$ is visible in the upper right. The text boxes are overlaid on this background.

Let's Recall!

Let's start by defining some terms:

- **Factors** are the numbers or polynomials that you multiply to form a product.
- A **polynomial** is factored when you write it as the product of two or more numbers or polynomials.
- A **polynomial** is factored completely when it is expressed as a product of one or more polynomials that cannot be factored further.
- A **polynomial** is prime if it has exactly two factors, itself and 1. The greatest common factor is the largest quantity that is a factor of all the integers or polynomials given.
- A **common monomial factor** is a number, a variable or the product of a number and a variable found in each term of the given polynomial.

Multiply each of the following.

1. $3(2x + 5)$

2. $m(3n - 8)$

3. $4x^2(6x^2 - 2x + 7)$

Concept

To factor polynomials with common monomial factor:

1. Find the greatest common factor (GCF) by factoring each term of the polynomial in its prime factors and getting all those factors that are found in all.
2. Divide each term of the polynomials by the greatest common monomial factor.
3. Multiply the greatest common monomial factor by the quotient obtained in step 2 to get the final answer.

Example 1:

Factor $5a - 5b + 10c$ completely.

Step 1: Find the GCF. The GCF is 5

Step 2: Divide each term of the polynomials by the greatest common monomial factor.

$$\frac{5a}{5} - \frac{5b}{5} + \frac{10c}{5} = a - b + 2$$

Step 3: Multiply the greatest common monomial factor by the quotient obtained in step 2 to get the final answer.

Final answer is $5(a-b+2c)$

Example 2: $5x + ax^2 + bx^3$ completely.

Step 1: Find the GCF.

Step 2: Divide each term of the polynomials by the greatest common monomial factor.

Step 3: Multiply the greatest common monomial factor by the quotient obtained in step 2 to get the final answer.

Example 3: $4a + 12a^2 + 20a^3$ completely.

Step 1: Find the GCF.

Step 2: Divide each term of the polynomials by the greatest common monomial factor.

Step 3: Multiply the greatest common monomial factor by the quotient obtained in step 2 to get the final answer.

Factor completely by filling the blanks.

1. $4a + 4b = \underline{\hspace{1cm}}(a + b)$

2. $ab + ac - ad = \underline{\hspace{1cm}}(b + c - d)$

3. $6x + 2x^2 - 8x^3 = \underline{\hspace{1cm}}(3 + x - 4x^2)$

4. $-12a + 9ab = -3a(\underline{\hspace{2cm}})$

5. $3x^3y + 2x^2y^2 - 6x^2y = x^2y(\underline{\hspace{4cm}})$

Factor each polynomial completely.

1. $12a - 12b$

2. $3x - 6y$

3. $4x^3 - 6x^2 - 10x$

4. $3x^3y - 4x^3y + 8x^2y^2$

5. $12x^2y^2 + 15xy^2 + 6x^2y$



TOPIC #2:

**FACTORING DIFFERENCE
OF TWO SQUARES**

Tell whether the following number is a perfect square or not.

1. 9

2. 25

3. 80

4. 144

5. 121

6. 225

7. 170

8. 100

9. $9y^2$

10. $4x^2$

Factoring Difference of Two Squares

The difference of two squares can be factored using this pattern: For any real numbers a and b ,

$$a^2 - b^2 = (a + b)(a - b).$$

Example: Factor the following expression.

1. $x^2 - 9$.

2. $16x^2 - 25y^2$

Complete the factors for each of the following.

1. $4x^2 - 49 = (2x + \underline{\quad})(2x - \underline{\quad})$

2. $36a^2 - b^4 = (\underline{\quad} + b^2)(\underline{\quad} - b^2)$

3. $64x^2y^2 - 121 = (\underline{\quad} + 11)\underline{\quad} - 11)$

4. $\frac{1}{16}m^2 - \frac{9}{25}m^2 = \left(\frac{1}{4}m + \underline{\quad}\right)\left(\frac{1}{4}m - \underline{\quad}\right)$

5. $0.09r^2 - 0.25s^2 = (0.3r + \underline{\quad})(0.3m - \underline{\quad})$

Factor each polynomial completely.

1. $a^2 - 25$

2. $9x^2 - 64$

3. $36a^2 - 1$

4. $49m^2 - 121n^4$

5. $100 - m^6n^8$

Operations on Integers

Addition and Subtraction:

1. $(+) + (+) = +$, $3 + 6 = 9$
2. $(-) + (-) = -$, $-3 + -7 = -10$
3. $(+) + (-)$ or $(-) + (+) =$ subtract and use the sign of the larger value, $4 + (-2) = 2$, $-8 + 3 = -5$
4. $(+) - (-) = (+)$, $5 - (-3) = 5 + (3) = 8$
5. $(-) - (+) = (-)$, $-15 - (9) = -15 + (-9) = -24$
6. $(-) - (-) = (-) + (+) =$ subtract and use the sign of the larger value, $-10 - (-3) = -10 + (3) = -7$

Operations on Integers

Multiplication and Division:

1. Same sign = (+)

Example: $8 \times 8 = 64$, $(-5) \times (-2) = 10$

2. Different Sign = (-)

Example: $9 \div (-3) = -3$, $(-42) \div 6 = 7$

Laws of Exponents

Law	Example
$x^1 = x$	$6^1 = 6$
$x^0 = 1$	$7^0 = 1$
$x^{-1} = 1/x$	$4^{-1} = 1/4$
$x^m x^n = x^{m+n}$	$x^2 x^3 = x^{2+3} = x^5$
$x^m / x^n = x^{m-n}$	$x^6 / x^2 = x^{6-2} = x^4$
$(x^m)^n = x^{mn}$	$(x^2)^3 = x^{2 \times 3} = x^6$
$(xy)^n = x^n y^n$	$(xy)^3 = x^3 y^3$
$(x/y)^n = x^n / y^n$	$(x/y)^2 = x^2 / y^2$
$x^{-n} = 1/x^n$	$x^{-3} = 1/x^3$

Find the product using FOIL Method.

1. $(x + 7)(x + 7)$

Find the product using FOIL Method.

1. $(x + 4)(x + 4)$

2. $(2x - 3)(2x - 3)$

3. $(x^2 + 5)(x^2 + 5)$

4. $(3y + 4)^2$

5. $(y^3 - 10)^2$

Factoring Perfect Square Trinomial

A Perfect Square Trinomial can be factored using this pattern: For any real numbers a and b ,

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Example: Factor the following expression.

1. $x^2 + 8x + 16$

2. $4m^2 - 12m + 9$

Remember:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Complete the trinomial to make it a perfect square trinomial.

1. $x^2 + \underline{\hspace{1cm}} + 64$

2. $a^2 - \underline{\hspace{1cm}} + 16b^2$

3. $y^2 - 12y + \underline{\hspace{1cm}}$

4. $\underline{\hspace{1cm}} + 20q^3 + q^6$

5. $9x^2 + \underline{\hspace{1cm}} + 16y^2$

Remember:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Factor each trinomial completely.

1. $x^2 + 6x + 9$

2. $1 - 2y + y^2$

3. $64a^2 - 16a + 1$

4. $16m^2 - 56m + 49$

5. $4x^2 + 20xy + 25y^2$

Evaluation

Answer the following.

1. What is the GCF of x^3 and x^6
2. What is the factored form of $16x^2 - 81$
3. Complete the factoring $6x^2 - 18x = 6x(\underline{\quad})$
4. Complete $4x^2 + (\underline{\quad}) + 9$ to make it a perfect square trinomial.
5. Factor $4x^2 - 20x + 25$.